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The Rol of Hybrid Numerical Methods in Fracture Mechanics

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ABSTRACT

This paper devotes at an attempt at reviewing the hybrid numerical methods developed in static and dynamic fracture mechanics. Moreover, in the framework of hybrid experimental-numerical methods, this paper provides a summary of our recent studies on simulation technologies for fracture path prediction. Recent successful examples of various fracture path prediction simulations are presented.
***Keywords**: Hybrid method, Mumerical method; Experimental method; Analytical method, Simulation technology.*

1. Introducere

Before the days of modern computer technology, we used to have only two major approaches for research and developments in the fields related to mechanics. The firs approach is „experimental methods", while the second one is „analytical methods" or „mathematical methods".
Now, „numerical methods" or „computational mechanics" can be considered as the third approach. Contrary to the firs and the second approaches, computational mechanics was originated by the progress of the computer technologies, about one decade to two decades ago.

Computational mechanics includes various numerical methodologies such as the finite difference method (FDM), the finite element method (FEM), the boundary element method (BEM) and the meshless methods.
All four approaches in mechanics have inherent advantages and disadvantages. In early days, the aforementioned numerical tools have acted as suppliment for the drawbacks of the experimental and mathematical approaches. However, recent rapid and great strides of computer technologies are producing borderless hybridization of the methods and approaches. Since most types of hybridization are based on digital computer technologies, most of these methods fall into the category of hybrid numerical method. The hybrid numerical methods infracture mechanics were firstly reviewed by Kobayashi [1]. Then, three types of hybrid methods of analysis were discussed by Atluri and Nishioka [2]. Later, the concepts of the hybrid numerical methods were clarified by Nishioka. On the basis of this concept, various hybrid numerical methods were clasified into five categoies [3]. Since the related literature is considerably vast and since the present author has been using various hybrid numerical methods in his research, in this paper, the hybrid numerical methods in static and dynamic fracture mechanics are reviewed.

2. Concept of hybrid numerical methods

For research and developments in mechanicas fields, experimental methods can be considered as the first approach. Experimental facts provide most important foundations for a related mechanics such as fracture mechanics. However, the experimental methods have several intrinsic drawbacks. For example, in most experimental methods, higher-order quantities such as various energy distributions are not directly measurable. Moreover, most expeimental measurements are done on the surfaces of bodies. Especially, it is not possible for the current experimental technologies alone to directly measure the mechanical quantities at the inside of an opaque material.

The second approach in mechanics is the analytical (mathematical) methods or theoretical mechanics. The analytical methods unify the experimental facts and establish the related theorie. However, most analytical methods cannot solve the problems with complex boundary conditions and those with some non-linearity.

The third approach, i.e., numerical methods or computaţional mechanics will become more powerful due to a promising future progress of computer ability. The past progress of computer ability has promoted the advanced developments of the convenţional numerical methods such as the FDM, FEM and BEM, individually. However, each numerical methodology has its own intrinsic drawbacks. In the third approach, new methodologies such as the neural networks, genetic algorithm and molecular dynamics are growing rapidly. These new types of numerical methods heavily rely on the abilities of computer, and provide the solutions which are not obtainable by the convenţional numerical methods. However, they are also not fully developed, and have their own intrinsic drawbacks.

In order to overcome the drawbacks in each method, and to produce a new advanced method, many types of hybridization of two or more than two totally different methods can be considered. Among these hybrid methods, any method that more or less relies on the numerical methods may be included in the category of the hybrid numerical methods. The hybrid numerical methods can be conceptually classified as shown in Fig. 1.

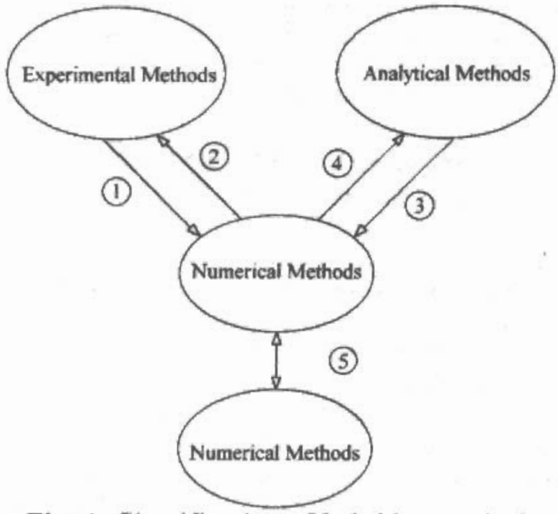


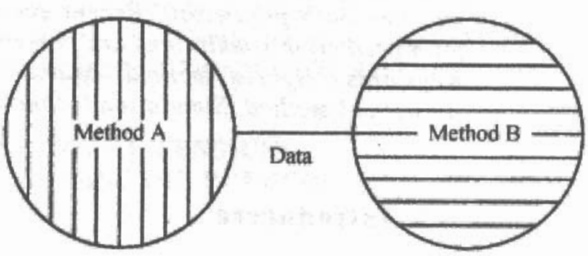
Fig. 1. Classification of hybrid numerical methods

Thus, the hybrid numerical methods are further divided as follows [3]:

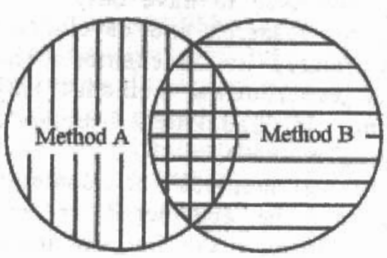
1. hybrid experimental-numerical methods;
2. hybrid numerical-experimental methods;
3. hybrid analytical-numerical methods;
4. hybrid numerical-analytical methods;
5. hybrid numerical-numerical methods.

2.1. State of hybridization

As described in the previous section, the hybrid method combines at least two totally different methods to produce a new ability. Let us consider the hybridization of two methods A and B. Depending on the state of connection of two methods, two types of the hybrid methods may be considered as depicted in Fig. 2. One of these is a *non-mixed hybrid method* in which the method A transfers necessary data to the method B. The method B gives the final solution of the problem considered. Means of the data transfer can be either *on-line* or *off-line*. In the off-line hybrid method, the methods A and B should be conceptually combined to lead a new solution technique for the problems which cannot be solved by the individual method alone.



a) Non-Mixed Hybrid Method



b) Mixed Hybrid Method

Fig. 2. States of connection in hybrid methods

The other one is a *mixed hybrid method* in which a part or the entire of the method A is mixed or overlapped with the method B. Each method may be either of software or hardware.

In the following subsections, several hybrid numerical methods are explained in further detail.

3. Hybrid analytical-numerical methods

In general an analytical solution expresses the essence of a given problem in a closed form, and conveniently provides the information on parametric behaviors of the solution of the problem. However, the analytical solutions that satisfy given governing equations are usually obtained for infinite or semi-infinite domains. On the other hand, general purpose numerical methods such as the FEM can obtain numerical solutions for arbitrary domains. However, to evaluate parametric behaviors of the solution of the problem, repeated numerical analyses are usually required.

In the applications of fracture mechanics, accurate evaluations of stress intensity factors (or the strengths of the singularities of the stress and strain states) are required along the borders of embedded or surface cracks in complex structural components. Nowadays, even though excellent results for the stress intensity factors can be obtained by the numerical methods such as the FEM and BEM, especially if path independent integrals are used, since the crack-front regions are explicitly modeled numerically, the number of algebraic equations is excessive, thus still rendering the ordinary numerical methods prohibitively expensive for some three-dimensional crack problems such as interacting multiple cracks.

To compensate the above disadvantages in the analytical solutions and the numerical solutions, various hybrid methods that combine both analytical and numerical techniques have been considered. These hybrid analytical-numerical methods may be classified as shown in Fig. 3, based on the level of analytical information introduced in the modeling.

The ordinary FEM as denoted by level 0 contains no analytical information for the considered problem and provides piecewise interpolated numerical solutions. Contrary to this, the Rayleigh-Ritz method (level 3) can use analytical solutions as the basis functions for the entire region. Although this method gives extremely good approximation around the crack tip region [4], it is difficult to satisfy the boundary conditions for complex specimen geometries.

The levels 1 and 2 belong to the category of the hybrid analytical-numerical method. Various singular element methods may be included in the level 1. In this level, the analytical solutions containing the $r^{-0.5}$ stress singularities are locally employed as the basis functions of element(s) adjacent to the crack tip. The singular elements are also called as special elements or crack-tip elements. Various types of singular elements are developed for static and dynamic crack problems. The review articles on the singular elements were given by several researchers (Atluri [5] for static crack problems, Nishioka and Atluri [6] for dynamic crack problems, Atluri and Nishioka [7] for 3D crack problems, etc.).

In the level 2, the analytical solutions are utilized for the entire regions or for part regions together with the FEM or BEM. The techniques in the level 2 can be classified into two typical methods; the superposition method and the alternating method.

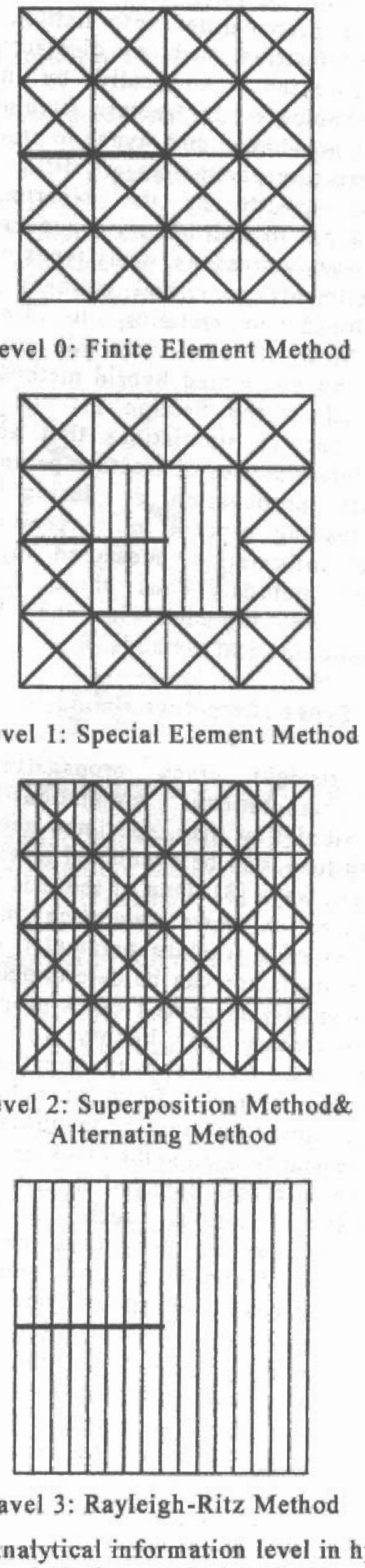


Fig. 3. Analytical information level in hybrid analytical-numerical methods

4. Hybrid experimental-numerical methods

Even using a full-field experimental measurement, the experimental method alone cannot give higher-order information on physical quantities of solid. In order to obtain more detailed higher-order information, the first-order information such as displacement must be processed numerically by using computer technologies. In fracture simulation, especially in non-linear and dynamic fracture simulation, fracture parameters are difficult to be extracted directly by the experimental method alone, due to their history dependence.

For the above reasons, many types of the hybrid experimental-numerical methods have been developed in relation to fracture mechanics. Most of these may fall into the category of the non-mixed hybrid method that was explained in the Section 2.1. In other words, the fracture simulations that aim to obtain accurate histories of fracture parameters during crack propagation or during time-dependent loading process must receive and follow the information measured by the experimental method. Thus those fracture simulations are included in the hybrid experimental-numerical methods.

4.1. Types of fracture simulation

For straight crack propagation (or growth), computational simulations with specified initial flaw size, specimen geometry, and applied load, can be conducted in either of two different ways [8]. One of these is the so-called *"generation phase simulation"* in which the variation of a fracture parameter such as stress-intensity factor can be determined, using an experimentally measured crack-propagation history (or crack-growth history) (a versus t curve or C versus t curve) as the input data into the computational model. From this calculation, one can determine the fracture toughness as the material resistance against the crack growth.

Once a crack-propagation criterion (or crack-growth criterion) such as relation between the fracture toughness and crack velocity in a dynamic fracture problem, is determined or postulated either numerically or experimentally, it may be used in the second type of computational simulation, the so-called *"application phase simulation"*. In this calculation, the crack propagation history (a versus t) can be determined by specifying the initial conditions and material fracture toughness data as inputs to the computational model. The application phase simulation is also sometimes called *"prediction"* or *"inverse"* simulation.

For curving crack growth, three types of numerical simulation can be considered as depicted in Fig. 4. First, the generation phase simulation can be conducted similarly with the generation phase simulation for straight crack growth, except additionally using experimental data on the curved crack-path history (see Fig. 4(i)). In the application phase simulation for curving crack growth, two criteria must be postulated or predetermined, as indicated in Fig. 4(ii). One of them is the crack-propagation criterion which is almost the same with the one used in the straight crack-growth simulation. However, the crack-propagation criterion for the curving crack growth may involve mixed-mode fracture parameters. Thus, it should be described by a fracture parameter taking into account mixed-mode conditions, such as the dynamic J integral (J') which was derived by Nishioka and Atluri [9]. The other one is a criterion for predicting the direction of crack propagation (propagation-direction criterion or growth-direction criterion).

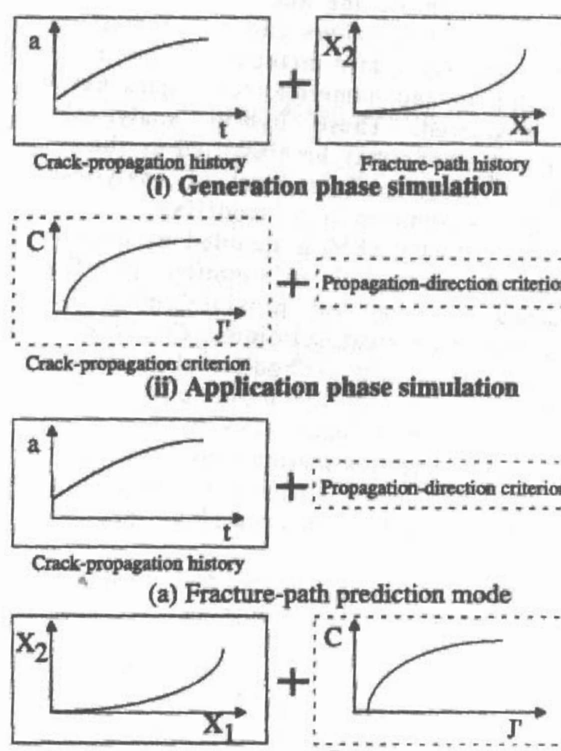


Fig. 4. Types of numerical simulation for fast curving crack propagation

The application phase simulations of curving crack growth have not fully developed except the application phase simulation of curving fatigue crack growth, due to several critical difficulties in those simulations for curving stable crack growth in elastic-plastic materials and for curving fast crack propagation in brittle materials. For instance, in dynamic brittle fracture, the crack-propagation criterion described by K_{ID} versus C relation itself has the unsolved problems as explained in the Section 4. Furthermore, the crack-propagation criteria may also be influenced by the geometries of fracture specimen.

In order to verify only the propagation-direction criterion such as the maximum energy release rate criterion, the present author proposes a *"mixed phase simulation"* as depicted in Figs. 4(iii)-(a). Regarding the crack-propagation history, the same experimental data for the a versus t relation used in the generation phase simulation can be used in the mixed-phase simulation. Thus, the increment of crack propagation is prescribed for the given time step sizes in the numerical simulation. Then the propagation-direction criterion predicts the direction of crack path in each time step. Simulated final fracture path will be compared with the experimentally obtained actual one. If the considered propagation-direction criterion is valid, the simulated fracture path should exactly agree with the actual one. Therefore, this mode of mixed-phase simulation may be called *"fracture-path prediction mode"*.

Another mode of mixed-phase simulation can be considered as depicted in Figs. 4(iii)-(b), i.e., *"crack-growth prediction mode"*. In this mode, the experimental data for the fracture-path history and the crack-propagation criterion are used simultaneously. Thus, the direction of crack propagation is prescribed at each time step. In this case, crack propagates along the actual fracture path during the numerical simulation. Simulated crack-propagation history should agree with the experimentally obtained actual one if the postulated crack-propagation criterion is valid.

5. Fracture-Path Prediction Procedures

Many propagation-direction criteria have been proposed in literature as listed below:

(i) maximum hoop stress criterion ($\sigma_{\theta\theta}$ max) (Erdogan and Sih [10]);

(ii) minimum strain energy density criterion (S-min) (Sih [11]);

(iii) maximum second stress invariant criterion (I_2 max) (Papadopoulos [12]),

(iv) maximum stress intensity factor criterion (K_I max) (Nemat-Nasser and Horii [13]),

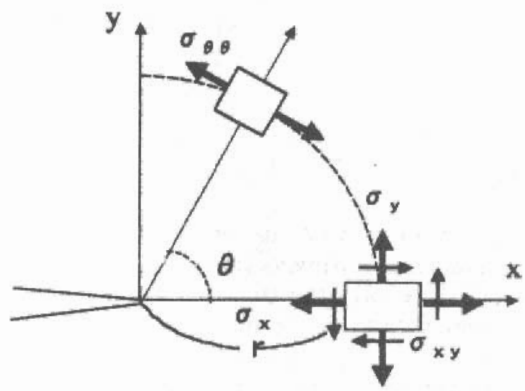
(v) maximum energy release rate criterion (G max) (Wu [14]),

(vi) local symmetry criterion (K_{II} =0) (Goldstein and Salganik [15]).

Recently Nishioka et al. [16] classified these criteria into two categories: (1) explicit prediction theory and (2) implicit prediction theory.

5.1. Explicit prediction theories

An explicit prediction theory predicts the propagation direction satisfying the postulated criterion based on a physical quantity for the current crack tip [16]. The criteria (i), (ii) and (iii) fall into this category. If the maximum hoop stress criterion is used, the crack is advanced in the direction of the maximum hoop stress (see Fig. 5), with a small crack-length increment ($\Delta a = C \cdot \Delta t$).



$$\sigma_{\theta\theta} = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\sigma_{xy} \sin\theta \cos\theta$$

Fig. 5. Explicit fracture path prediction procedure based on the maximum hoop stress criterion

5.2. Implicit prediction theories

Contrary to this, an implicit prediction theory seeks the propagation direction that satisfies the postulated criterion based on a physical quantity after the crack is advanced with a small crack-length increment [16]. An iterative process is generally needed to find the propagation direction. The criteria (iv), (v) and (vi) are classified into this category. It is known that in general the implicit prediction theories are more accurate.

Nishioka et al. [7, 8] have succeeded in predicting a smooth fast curving fracture path in a DCB specimen, using the moving isoparametric element method based on the mapping technique. In these studies, the criteria (iv)-(vi) were tested in the mixed-phase simulation with the fracture-path prediction mode. It was found from these simulations that the predicted fracture path based on the local symmetry criterion (K_{II}=0) most accurately agreed with the actual experimental fracture path. Thus, in this subsection, numerical procedures based on only the local symmetry criterion are explained.

Figure 6 schematically explains the numerical procedures for the path-prediction mode of the mixed-phase simulation. In each time step, the crack is advanced by a small increment according to the experimental history (crack-length versus time curve).

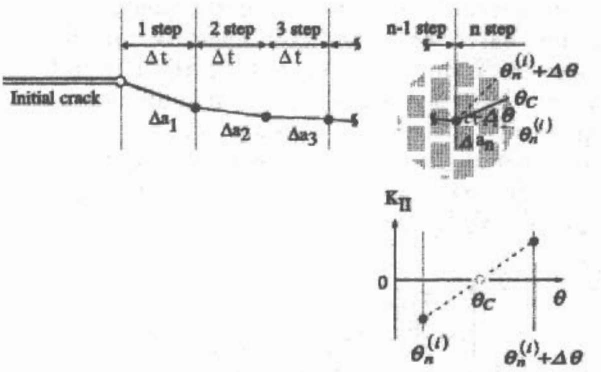


Fig. 6. Implicit fracture path prediction procedure based on the local symmetry criterion

The fracture path is predicted in an iterative manner as follows: In the following a superscript (i) denotes the iteration number. At a generic time step n, as the first trial, the crack is advanced in the tangential direction $\theta^{(i)}$ (i=1) at the crack tip of the step n-1. If an employed propagation-direction criterion, for example the local symmetry (K_{II}=0) criterion, is satisfied at the attempted crack-tip location, the crack is advanced in this direction, $\theta^{(i)}$. If the K_{II} value is negative, the crack is tentatively advanced in the direction of $\theta^{(i+1)}=\theta^{(i)}+\Delta\theta$ as the next trial. If the K_{II} value is positive, $\Delta\theta$ is taken as negative. Then, the satisfaction of the criterion at the trial crack tip location is checked. If the criterion is not satisfied and the value of the K_{II} value at the present trial is different with that of the previous trial, the next trial direction $\theta_n^{(i+1)}$ that satisfies the employed propagation-direction criterion is predicted by the K_{II} versus θ curve as shown in Fig. 6. If the predicted angle θ_c largely differs from the trial angle of the current iteration $\theta_n^{(i)}$, the next trial direction is taken as $\theta_n^{(i+1)}=\theta_n^{(i)}+\Delta\theta$. The crack is advanced in this direction and the satisfaction of the criterion is checked. These procedures are repeated until the criterion is satisfied. At each iteration, remeshing is needed if the implicit criterion is used, as explained above. After finding the propagation direction that exactly satisfies the postulated criterion, the time step proceeds to the next step.

6. Various Fracture Path Prediction Simulations

Using various moving finite element methods, Nishioka and coworkers have carried out various types of dynamic fracture simulations. These simulation results were timely summarized in review articles on the theoretical and computational aspects of dynamic fracture (Nishioka and Atluri [19], Nishioka [20, 21-26]). In this section, recent successful examples of various fracture path prediction simulations are explained.

5.1. Fracture path prediction in materials containing inclusions and voids

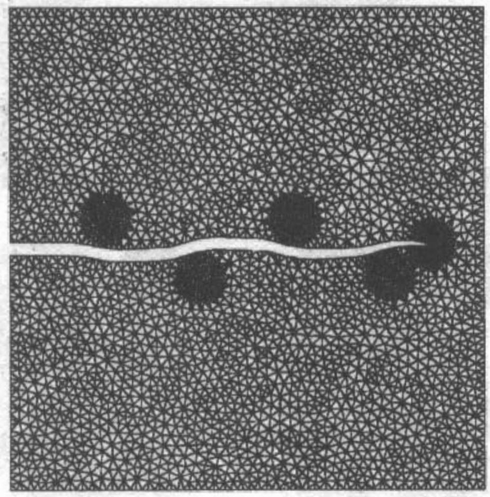
For elastic materials containing inclusions and voids, mesoscale stable fractures were considered, using periodic condition for a unit cell (4mm×2mm). The base material is considered to be PMMA (Young's modulus E=2.95GPa, Poisson's ratio ν=0.33) while the inclusion is modeled as a mild steel (E=206GPa, ν=0.28). The application phase simulations were carried out based on a critical K_{IC} value for the crack propagation criterion together with the local symmetry criterion (K_{II}=0) for the propagation direction criterion. In each step the crack was advanced by the increment of 20μm. Displacement controlled loads were applied at the top and bottom ends of the unit cell.

Figure 7 shows the predicted fracture path in the right half part of the unit cell from the initial center crack ($2a_0$=200μm) (see the left straight portion of the crack). In the material containing inclusions, the crack tends to propagate avoiding the inclusions. It was found in other simulations, the curving becomes smaller when the Young's modulus of the inclusion decreases. Obviously the crack propagates straight in a homogeneous material. In the material containing voids, the crack tends to propagate toward the voids. If the vertical spacing of the voids is closer, the crack may propagate into one of the voids.

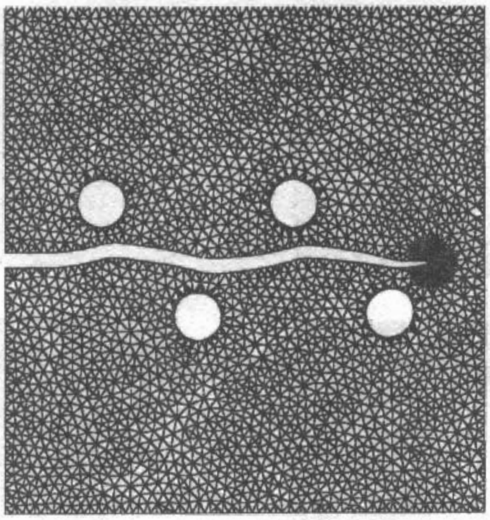
6.2. Dynamic kinking fracture from an interface crack

First, an experimental was carried out for the interfacial crack kinking phenomenon in a bimaterial specimen of epoxy and aluminum alloy [27]. Under the static loading angle of 120° measured anticlockwise from the interface line, the fractured specimen was obtained as shown in Fig. 8(a). The crack kinked from the initial interface crack tip and propagated dynamically in the epoxy side.

Using the maximum energy release rate criterion, the mixed-phase fracture path prediction mode simulation was carried out. The simulation result is shown in Fig. 8(b). The numerically predicted fracture path agrees excellently with the experimental fracture path.

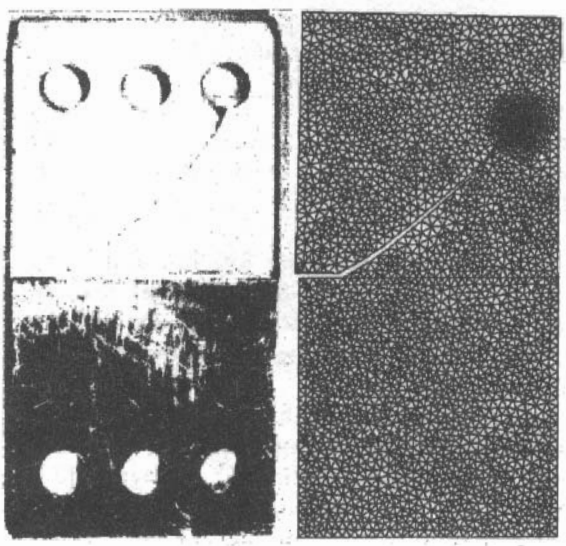


a) inclusions



b) voids

Fig. 7. Curving fracture in mesoscale materials



a) experiment b) simulation

Fig. 8. Dynamic interfacial crack kinking

6.3. Mixed-Phase Fracture-Path Prediction Mode Simulation of Impact Mixed-Mode Fracture

Three-point bend specimens of PMMA (Polymethyl Methacrylate) were used for mixed-mode impact fracture test [28]. The geometry of the impact fracture specimen is shown in Figure 9. The initial crack was placed along the center of the specimen. The impact load by a dropping rod (5.05 kg) was applied at an off-center point, to induce a mix-mode state at the initial crack tip. The impact velocity of the rod was set as 5m/s. The loading eccentricity is defined as $e=l/(S/2)$ where l is the distance between the loading point and the centerline of the specimen, and S is the span of the supports of the specimen.

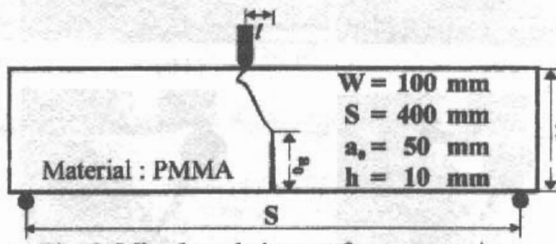


Fig. 9. Mixed-mode impact fracture specimen

The high-speed photographs of dynamically fracturing specimen under the loading eccentricity of $e=0.1$ are shown in Figure 20. The time intervals were about 30μs. In the photographs (4), (5) and (6), the caustic patterns of Mode II dominated type can be seen. At the time of the photograph (7), the crack had already propagated for a short distance. The caustic pattern in the photograph (7) suddenly changed to Mode I dominated type. The crack started propagating at $t=120$ μs after the initiation of impact ($t=0$). The maximum crack velocity observed was 300 m/s.

To simulate dynamic kinking and curving fracture phenomena, the moving finite element method based on Delaunay automatic mesh generation was developed by Nishioka, Tokudome and Kinoshita, [29, 30].

Using the local symmetry ($K_{II}=0$) criterion together with the experimentally obtained crack-propagation history, the mixed-phase fracture-path prediction mode simulation was carried out.

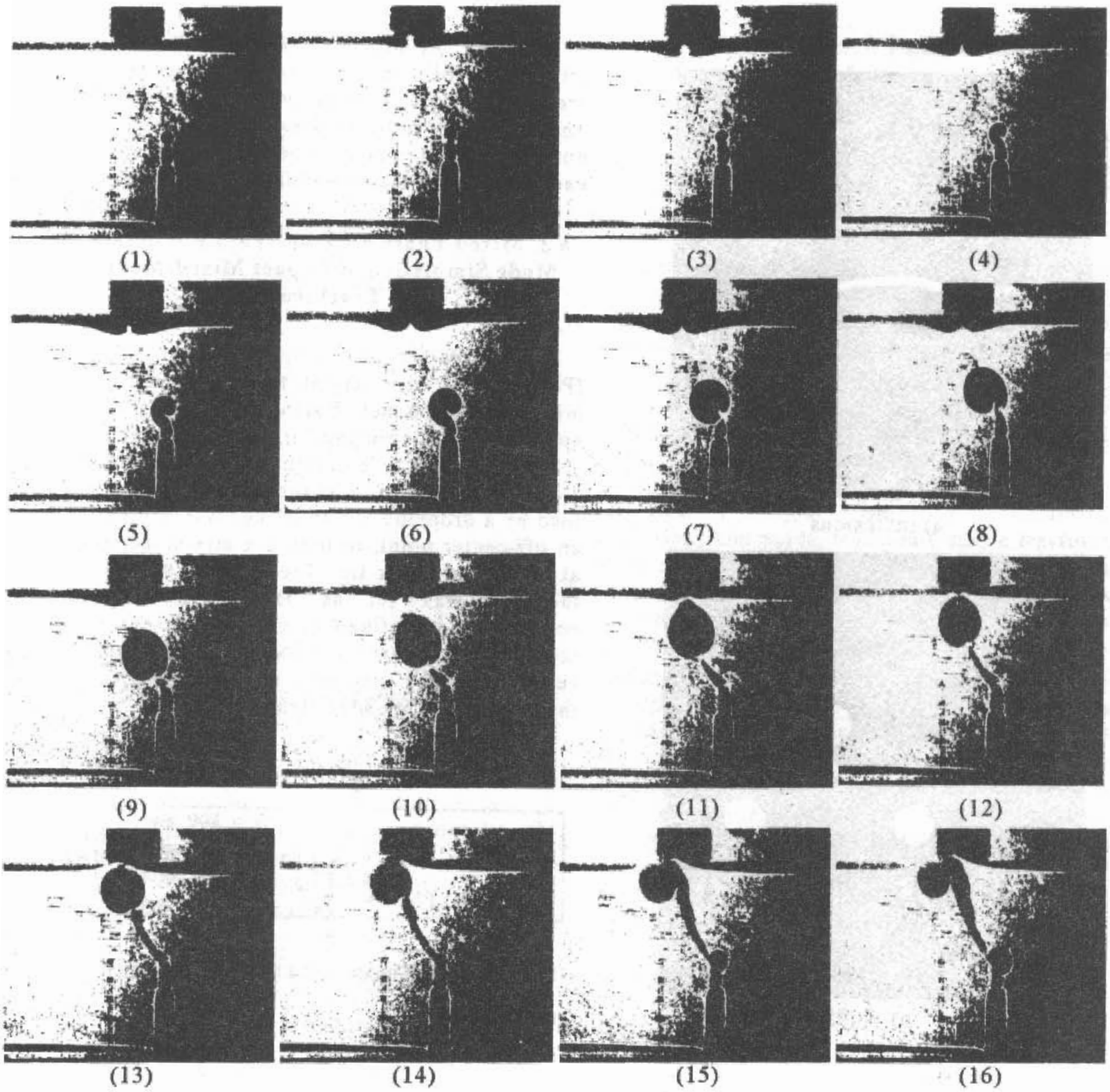


Fig. 10. High-speed photographs of mixed-mode impact fracture (time intervals: 30μs)

The simulated fracture path and the overall deformation of the dynamically fracturing specimen are shown in Figure 11. The deformation is magnified by 30 times. It is seen that the postulated local symmetry criterion predicts the dynamically propagating fracture path toward the point of impact loading. The simulated fracture path excellently agrees with the experimental fracture path (see Figure 10).

6.4. Elastic-plastic curving stable fracture

To establish an accurate simulation method for complex crack propagation in nonlinear materials, first, Nishioka, Kobayashi and Fujimoto [31] derived an incremental variational principle to satisfy the boundary conditions near newly created crack surfaces. Based on this variational principle the moving finite element method for nonlinear fracture was developed.

Curving fracture tests for thin aluminum alloy plates under mixed-mode biaxial loading were performed by Lam et al. [32]. One of the tests was simulated using the moving finite element method.

The fracture path prediction mode simulation was carried out based on the local symmetry criterion of the T* integral [33] ($T_2^{*0}=0$). The predicted fracture path is shown in Fig. 12. The fracture path agrees excellently with the experimentally observed one.

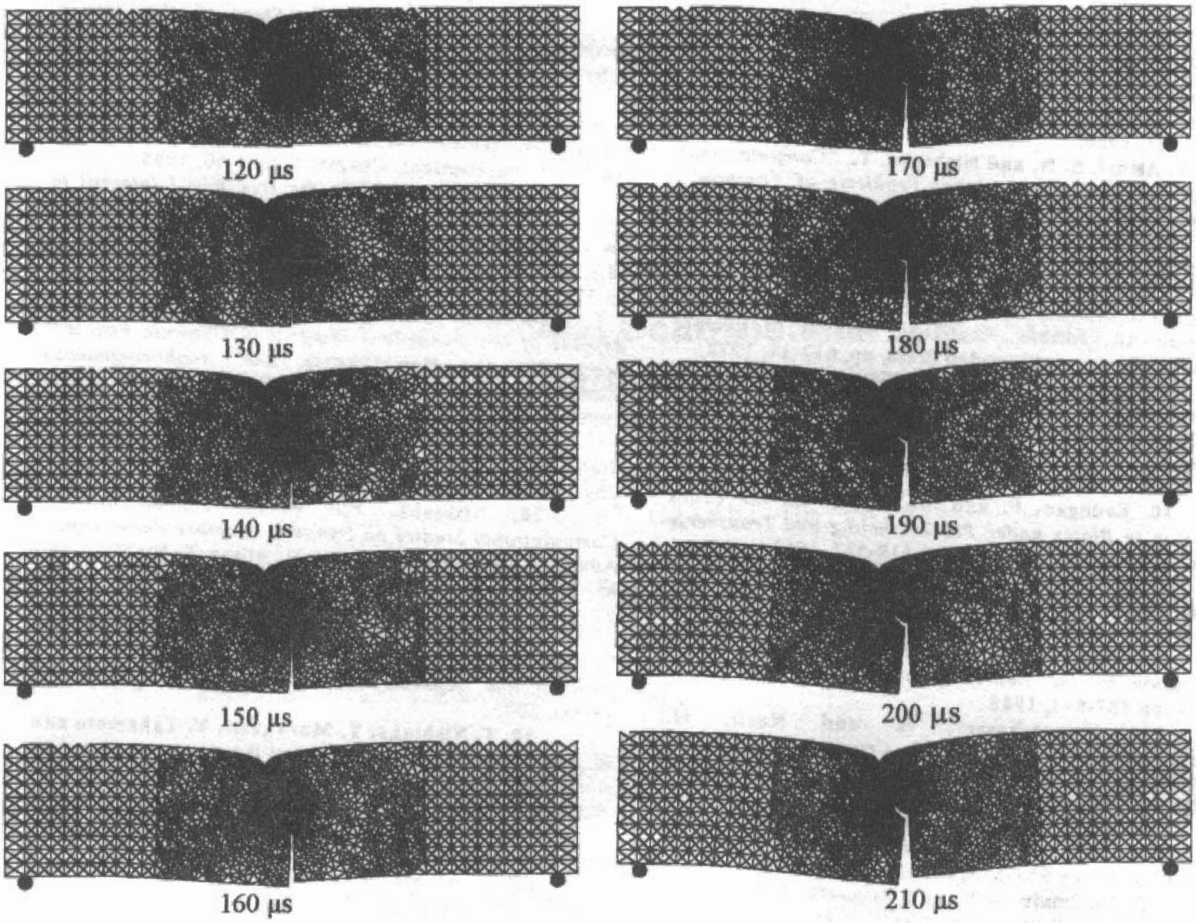


Fig. 11. Simulation results for dynamic fracture path prediction with the K_{II}=0 criterion (e=0.1)

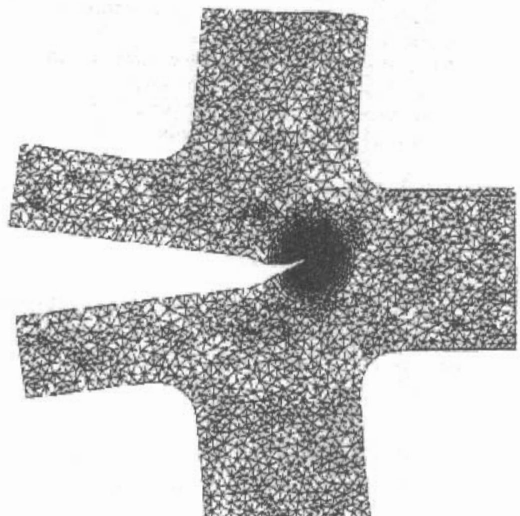


Fig. 12. Elastic-plastic curving fracture

7. Concluding remarks

In this paper, the concept of hybrid numerical methods was presented. Based on this concept, various hybrid numerical methods used in static and dynamic fracture mechanics were classified into five categories. A special attention is paid to the hybrid-numerical methods. In the framework of these methods, the fracture-path prediction procedures were explained in detail.

The numerical simulations presented in this paper provided valuable results and insights to clarify the mechanisms of various dynamic fracture phenomena.

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Le Rol des Méthodes Numériques Hybrides dans la Mécanique de Rupture

Cet article consacre à une tentative à passer en revue les méthodes numériques hybrides développées dans la mécanique statique et dynamique de rupture. D'ailleurs, dans le cadre des méthodes numériques expérimentales hybrides, cet article fournit un sommaire de nos études récentes sur des technologies de simulation pour la prévision de chemin de rupture. Des exemples réussis récents de diverses simulations de prévision de chemin de rupture sont présentés.